LIMITI – ESERCIZI CON SOLUZIONI

Calcolare il valore dei seguenti limiti:

1) $\lim_{x \to 3} \frac{x-1}{x^2+2x-5} = \frac{3-1}{3^2+2\cdot 3-5} = \frac{2}{10} = \frac{1}{5}$

2) $\lim_{x \to 0^+} \frac{1}{x(x+2)} = +\infty$ PERCHÈ $x(x+2) \to 0^+ \Rightarrow \frac{1}{x(x+2)} \to +\infty$

3) $\lim_{x \to 2^-} \frac{x}{x-2} = -\infty$ PERCHÈ $x-2 \to 0^-$

4) $\lim_{x \to -1^+} \frac{1-x}{x+1} = +\infty$ PERCHÈ $x+1 \to 0^+$ E $1-x \to +2$

5) $\lim_{x \to -1^-} \frac{1-x}{x+1} = -\infty$ PERCHÈ $x+1 \to 0^-$ E $1-x \to +2$

6) $\lim_{x \to 0} \log(x^2-3x+1) = \log 1 = 0$

7) $\lim_{x \to +\infty} \frac{1}{\log_2 x} = 0$ PERCHÈ $\log_2 x \to +\infty$

8) $\lim_{x \to 0^+} \frac{1}{\log_2 x} = 0$ PERCHÈ $\log_2 x \to -\infty$

9) $\lim_{x \to e^+} \ln(x-e) = -\infty$ PERCHÈ $x-e \to 0^+$

10) $\lim_{x \to 1} \log 10^x = \log 10^1 = 1$

11) $\lim_{x \to +\infty} e^{x+1} = +\infty$

12) $\lim_{x \to +\infty} \left(\frac{1}{2}\right)^{x-1} = 0$

13) $\lim_{x \to -\infty} e^{x+1} = 0$

14) $\lim_{x \to 3} 2^x = 2^3 = 8$

15) $\lim_{x \to 1^+} e^{\log(x-1)} = 0$ PERCHÈ $x-1 \to 0^+ \Rightarrow \log(x-1) \to -\infty$

16) $\lim_{x \to -\infty} 5 = 5$

17) $\lim_{x \to 3} 5 = 5$

18) $\lim_{x \to +\infty} 5 = 5$

IL LIMITE DI UNA FUNZIONE COSTANTE È SEMPRE UGUALE AL VALORE DELLA FUNZIONE

19) $\lim_{x \to +\infty} (x^2 - x + 1) =$ / FORMA INDETERMINATA DEL TIPO $+\infty - \infty$ /

$$= \lim_{x \to +\infty} x^2\left(1 - \underbrace{\frac{1}{x}}_{\to 0} + \underbrace{\frac{1}{x^2}}_{\to 0}\right) = +\infty$$

20) $\lim_{x \to -\infty} (x^3 + 2x^2) =$ / FORMA INDETERMINATA DEL TIPO $+\infty - \infty$ /

$$= \lim_{x \to -\infty} x^3\left(1 + \underbrace{\frac{2}{x}}_{\to 0}\right) = -\infty$$

21) $\lim_{x \to +\infty} (2x^3 - x^2 + 5x + 7) =$ / FORMA INDETERMINATA DEL TIPO $+\infty - \infty$ /

$$= \lim_{x \to +\infty} x^3\left(2 - \underbrace{\frac{1}{x}}_{\to 0} + \underbrace{\frac{5}{x^2}}_{\to 0} + \underbrace{\frac{7}{x^3}}_{\to 0}\right) = +\infty$$

22) $\lim_{x \to +\infty} \frac{x^2 - x + 1}{2x - 3} =$ / FORMA INDETERMINATA DEL TIPO $\frac{\infty}{\infty}$ /

$$= \lim_{x \to +\infty} \frac{x^{\cancel{2}}\left(1 - \underbrace{\frac{1}{x}}_{\to 0} + \underbrace{\frac{1}{x^2}}_{\to 0}\right)}{\cancel{x}\left(2 - \underbrace{\frac{3}{x}}_{\to 0}\right)} = +\infty$$

23) $\lim_{x \to -\infty} \frac{3x^2-2x+5}{7x^2-1}$ = / FORMA INDETERMINATA DEL TIPO $\frac{\infty}{\infty}$ /

$$= \lim_{x \to -\infty} \frac{\cancel{x^2}\left(3-\frac{2}{x}+\frac{5}{x^2}\right)}{\cancel{x^2}\left(7-\frac{1}{x^2}\right)} = \frac{3}{7}$$

24) $\lim_{x \to +\infty} \frac{x}{x^2+4x-1}$ = / FORMA INDETERMINATA DEL TIPO $\frac{\infty}{\infty}$ /

$$= \lim_{x \to +\infty} \frac{\cancel{x}}{x^{\cancel{2}}\left(1+\frac{4}{x}-\frac{1}{x^2}\right)} = 0$$

25) $\lim_{x \to -\infty} e^{\frac{x-1}{x^2+1}}$ = / FORMA INDETERMINATA DEL TIPO $\frac{\infty}{\infty}$ /

$$= \lim_{x \to -\infty} e^{\frac{\cancel{x}\left(1-\frac{1}{x}\right)}{x^{\cancel{2}}\left(1+\frac{1}{x^2}\right)}} = e^0 = 1$$

26) $\lim_{x \to +\infty} \log\left(\frac{x^3-1}{x^3-5x}\right)$ = / FORMA INDETERMINATA DEL TIPO $\frac{\infty}{\infty}$ /

$$= \lim_{x \to +\infty} \log \frac{\cancel{x^3}\left(1-\frac{1}{x^3}\right)}{\cancel{x^3}\left(1-\frac{5}{x^2}\right)} = \log 1 = 0$$

27) $\lim_{x \to 3} \frac{x^2-9}{x-3}$ = / FORMA INDETERMINATA DEL TIPO $\frac{0}{0}$ /

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = 6$$ SI È FATTORIZZATO IL NUMERATORE

28) $\lim_{x \to -1} \frac{x^2+2x+1}{x+1}$ = / FORMA INDETERMINATA DEL TIPO $\frac{0}{0}$ /

$$= \lim_{x \to -1} \frac{(x+1)^{\cancel{2}}}{\cancel{x+1}} = 0$$ SI È FATTORIZZATO IL NUMERATORE

29) $\lim_{x\to 0^+} \frac{\sqrt{x}}{x} =$ / FORMA INDETERMINATA DEL TIPO $\frac{0}{0}$ /

$= \lim_{x\to 0^+} \frac{\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}\cdot\sqrt{x}} = +\infty$ SI È FATTORIZZATO IL DENOMINATORE

LIMITI CHE SI RISOLVONO CON IL LIMITE NOTEVOLE $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

30) $\lim_{x\to+\infty}\left(1-\frac{1}{x}\right)^x = \lim_{x\to+\infty}\left(1+\frac{1}{(-x)}\right)^{(-x)\cdot(-1)} \underset{\uparrow}{=} \lim_{t\to-\infty}\left[\left(1+\frac{1}{t}\right)^t\right]^{-1} = e^{-1} = \frac{1}{e}$

SI PONE $-x=t \Rightarrow t\to-\infty$

31) $\lim_{x\to+\infty}\left(1+\frac{3}{x}\right)^{5x} = \lim_{x\to+\infty}\left(1+\frac{1}{\frac{x}{3}}\right)^{\frac{x}{3}\cdot 15} \underset{\uparrow}{=} \lim_{t\to+\infty}\left[\left(1+\frac{1}{t}\right)^t\right]^{15} = e^{15}$

SI PONE $\frac{x}{3}=t \Rightarrow t\to+\infty$

32) $\lim_{x\to 0^+}(1+2x)^{\frac{1}{x}} = \lim_{x\to 0^+}(1+2x)^{\frac{1}{2x}\cdot 2} \underset{\uparrow}{=} \lim_{t\to+\infty}\left[\left(1+\frac{1}{t}\right)^t\right]^2 = e^2$

SI PONE $t=\frac{1}{2x} \Rightarrow 2x=\frac{1}{t}$ e $t\to+\infty$

33) $\lim_{x\to 0}\frac{e^x-1}{x} \underset{\uparrow}{=} \lim_{t\to 0}\frac{t}{\ln(t+1)} = \lim_{t\to 0}\frac{1}{\frac{1}{t}\ln(t+1)} = \lim_{t\to 0}\frac{1}{\ln\underbrace{(t+1)^{\frac{1}{t}}}_{e}} = \frac{1}{\ln e} = 1$

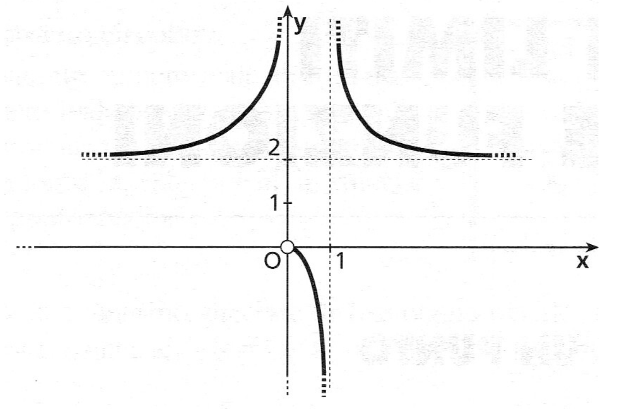
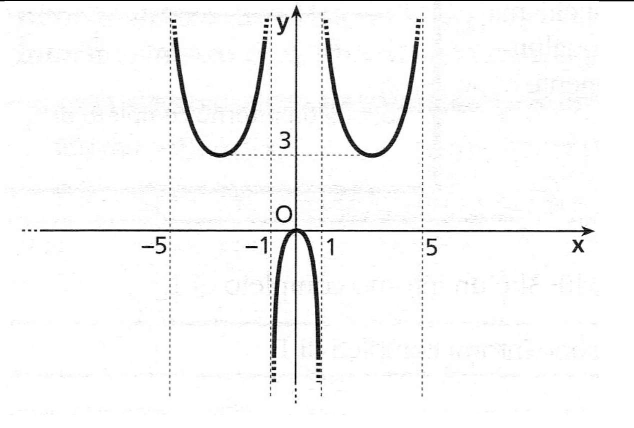
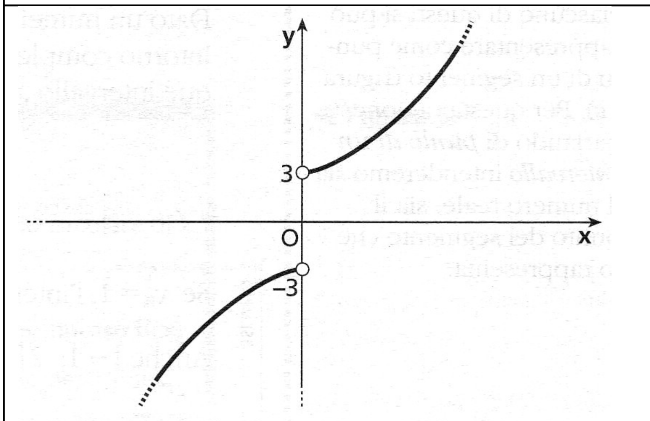
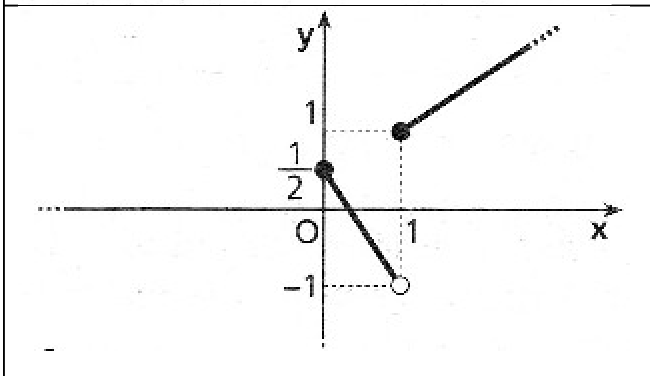
SI PONE $e^x-1=t \Rightarrow e^x=t+1 \Rightarrow x=\ln(t+1)$ e $t\to 0$

34) $\lim_{x\to+\infty}\left(\frac{x-2}{x-3}\right)^{x-3} = \lim_{x\to+\infty}\left(\frac{x-3+1}{x-3}\right)^{x-3} = \lim_{x\to+\infty}\left(\frac{x-3}{x-3}+\frac{1}{x-3}\right)^{x-3} =$

$= \lim_{x\to+\infty}\left(1+\frac{1}{x-3}\right)^{x-3} \underset{\uparrow}{=} \lim_{t\to+\infty}\left(1+\frac{1}{t}\right)^t = e$

SI PONE $x-3=t \Rightarrow t\to+\infty$

Calcolare il dominio delle seguenti funzioni e i limiti agli estremi del dominio:

	$D=(-\infty;0)\cup(0;1)\cup(1;+\infty)$ $\lim_{x\to-\infty} f(x)=2$ $\lim_{x\to0^-} f(x)=+\infty$ $\lim_{x\to0^+} f(x)=0$ $\lim_{x\to1^-} f(x)=-\infty$ $\lim_{x\to1^+} f(x)=+\infty$ $\lim_{x\to+\infty} f(x)=2$
	$D=(-5;-1)\cup(-1;1)\cup(1;5)$ $\lim_{x\to-5^+} f(x)=+\infty$ $\lim_{x\to-1^-} f(x)=+\infty$ $\lim_{x\to-1^+} f(x)=-\infty$ $\lim_{x\to1^-} f(x)=-\infty$ $\lim_{x\to1^+} f(x)=+\infty$ $\lim_{x\to5^-} f(x)=+\infty$
	$D=(-\infty;0)\cup(0;+\infty)$ $\lim_{x\to-\infty} f(x)=-\infty$ $\lim_{x\to0^-} f(x)=-3$ $\lim_{x\to0^+} f(x)=3$ $\lim_{x\to+\infty} f(x)=+\infty$
	$D=[0;+\infty)$ $\lim_{x\to0^+} f(x)=\frac{1}{2}$ $\lim_{x\to1^-} f(x)=-1$ $\lim_{x\to1^+} f(x)=1$ $\lim_{x\to+\infty} f(x)=+\infty$